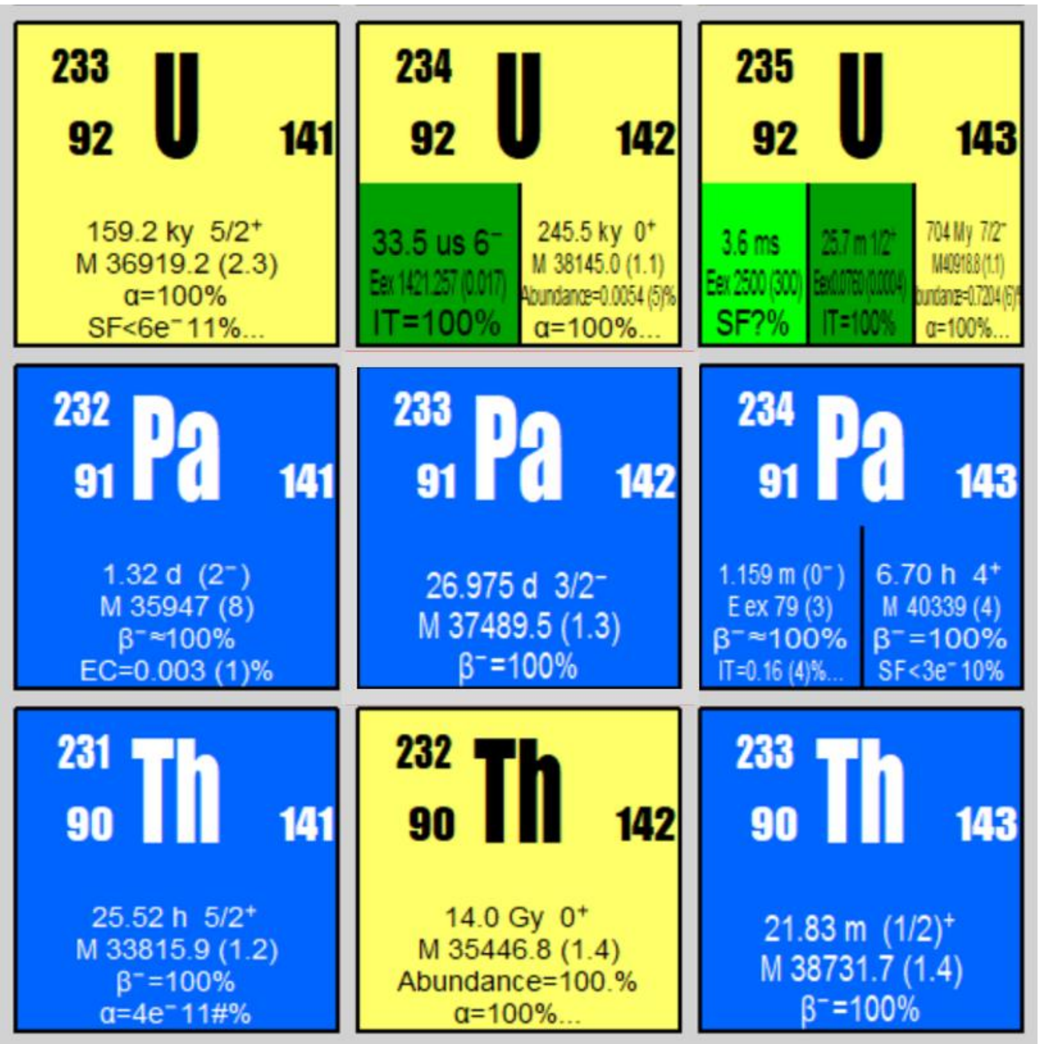
Read the questions carefully and mind the units.

Write your name and student number. This exam comprises 3 problems. The total number of points is 30. The final grade is obtained by dividing the number of points by 3.

Problem 1 (8 pts)

Thorium is a fertile material. However, upon the absorption of a neutron it can transform into a fissile nuclide.

a. Use the chart of nuclides below to show that Th-232 can transform into U-233. (4 pts)



b. Natural uranium is able to sustain a chain reaction, as is illustrated by the CANDU reactors. This is not the case for thorium. Explain how thorium can be used as fuel for a fission reactor. (4 pts)

Solutions

a. $n + {}^{232}_{90}Th \longrightarrow {}^{233}_{90}Th \xrightarrow{\beta^-} {}^{233}_{91}Pa \xrightarrow{\beta^-} {}^{233}_{92}U$

b. Thorium must be mixed with a fissile material (fissile driver), such as U-233, U-235 or Pu-239.

Problem 2 (12 pts)

Consider the reaction in a fusion reactor:

$$ {}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n $$

a. Calculate the energy in MeV released by this reaction. Compare this result with the average energy produced by a U-235 fission reaction and deduce which reaction releases the most energy. (4 pts)

b. Assume now equal weights of fuel for the fusion and fission reactions. Which reaction releases the most energy? (4 pts)

c. The D-T fusion produces a helium nucleus and a neutron. Explain why one particle escapes the plasma of the fusion reactor, while the other one stays inside and how the energy of the fusion reaction is harnessed to eventually produce electrical power. (4 pts)

Element	Mass excess (keV)
n	8071.3171
^{2}H	13135.72176
^{3}H	14949.80993
^{4}He	2424.91561

Solutions

a. We first calculate the Q-value of the D-T reaction:

$$\begin{aligned} Q &= M(D) + M(T) - M\left({}^{4}He\right) - M(n) \\ &= 13135.72176 + 14949.80993 - 2424.91561 - 8071.3171 \\ &= 17589.3\ \text{keV} = 17.59\ \text{MeV} \end{aligned}$$

The average energy released by the fission of U-235 is 200 MeV.
It is clear that the fission reaction releases more energy than the fusion one.

b. For equal weights of fuel for both reactions, we need to calculate the energy released per nucleon.

Since five nucleons participate in the fusion reaction, the energy released per nucleon is

$$\frac{17.59}{5} = 3.52\ MeV/nucleon$$

The average energy released by the fission of U-235 is 200 MeV. This reaction involves 236 nucleons $\left({}^{235}_{92}U + n \rightarrow {}^{236}U^*\right)$. The energy per nucleon released in a U-235 fission reaction is

$$\frac{200}{236} = 0.85\ MeV/nucleon$$

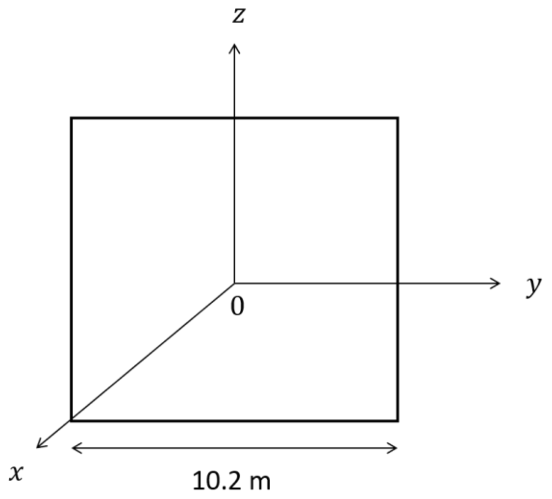
Therefore, for equal weights of fuel, the fusion energy releases more energy.

c. The neutron escapes the plasma since it is uncharged and is not influenced by the magnetic fields. The helium nucleus is positively charged and is confined in the plasma. The neutron is stopped by the blanket where it deposits its energy, which is carried away by water flowing through (heat exchanger).

Problem 3 (12 pts)

A research reactor operates at a steady power of 24 MW and consists of a cubical array of natural uranium rods in a graphite moderator. Each side of the cube is 10.2 m long. The average value of the macroscopic fission cross section is $\Sigma_f = 2.4\ 10^{-3}\ cm^{-1}$.
The average number of neutrons produced per fission ν is 2.42 and the diffusion coefficient D is 1.152 cm.

a. Calculate the buckling (B^2). (2 pts)
b. Calculate the average value of the neutron flux. (2 pts)
c. We set the coordinates of the centre of the cube reactor at (0, 0, 0) as seen in the figure below. Calculate the ratio of the flux on the z-axis at a distance of 5 m from the centre of the reactor to the flux on the z-axis at a distance of 0.1 m from the centre. (2 pts)
d. Does the previous result in c. make sense? Justify your answer. (2 pts)
e. Calculate the average fission rate in the entire reactor in nuclides/s. (2 pts)
f. Calculate the macroscopic absorption cross section. (2 pts)



Solution

a. The buckling for a cube is

$$B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = 3 \times \left(\frac{\pi}{a}\right)^2 = 3 \times \left(\frac{\pi}{1020}\right)^2 = 2.85\ 10^{-5}\ cm^{-2}$$

b. The average power in the reactor is given by

$$P = E_f R_f V = E_f \Sigma_f \phi V \Longrightarrow \phi = \frac{P}{E_f \Sigma_f V}$$

$$\phi = \frac{24\ MW}{200\ MeV \times 1.6\ 10^{-13} \times 2.4\ 10^{-3} cm^{-1} \times (1020\ cm)^3} = 2.94\ 10^{11} cm^{-2} \cdot s^{-1}$$

c. The flux at any point in this cube reactor is given by

$$\phi(x, y, z) = A\ \cos\left(\frac{\pi}{a} x\right) \cos\left(\frac{\pi}{a} y\right) \cos\left(\frac{\pi}{a} z\right)$$

The flux at 5 m on the z-axis from the centre is

$$\phi(0, 0, 500) = A\ \cos\left(\frac{\pi}{1020} 0\right) \cos\left(\frac{\pi}{1020} 0\right) \cos\left(\frac{\pi}{1020} 500\right)$$

The flux on the z-axis at 0.1 m from the centre of the cube is

$$\phi(0, 0, 10) = A\ \cos\left(\frac{\pi}{1020} 0\right) \cos\left(\frac{\pi}{1020} 0\right) \cos\left(\frac{\pi}{1020} 10\right)$$

The ratio is given by

$$\frac{\phi(0, 0, 500)}{\phi(0, 0, 10)} = \frac{\cos\left(\frac{\pi}{1020} 500\right)}{\cos\left(\frac{\pi}{1020} 10\right)} = 0.031$$

d. The result in c. shows that the neutron flux at 5 m from the centre is much less intense than close to the centre. This makes sense since the maximum flux is always at the centre of the reactor.

e. The fission rate in the entire reactor is

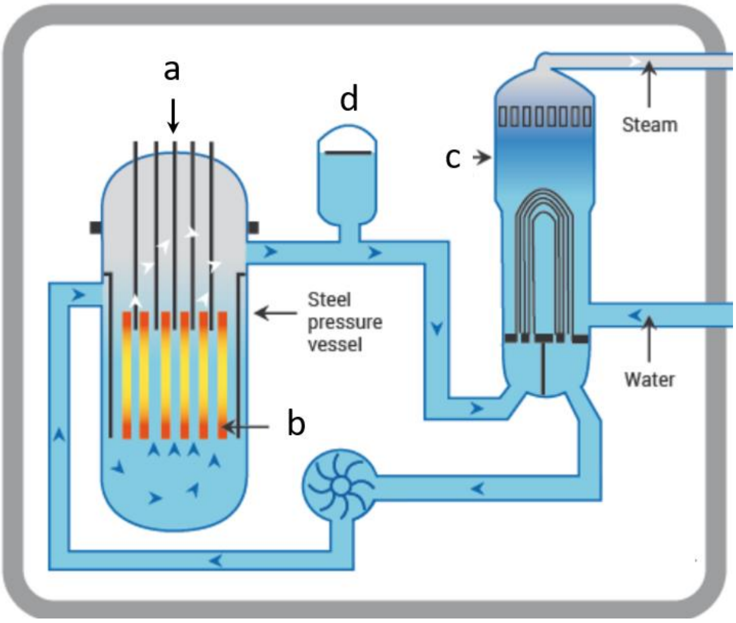
$$\Sigma_f \phi V = 2.4\ 10^{-3}\ cm^{-1} \times 5.05\ 10^{11}\ cm^{-2} \cdot s^{-1} \times (1020\ cm)^3 = 1.3 \times 10^{18}\ s^{-1}$$

f. The reactor is operating in steady state conditions, therefore, $k = 1$.

$$k = \frac{\nu\ \Sigma_f}{\Sigma_a + DB^2} = 1 \Longrightarrow \Sigma_a = \nu\ \Sigma_f - DB^2$$

$$\Sigma_a = 2.42 \times 2.4\ 10^{-3}\ cm^{-1} - 1.152\ cm \times 2.85\ 10^{-5}\ cm^{-2} = 0.0058\ cm^{-1}$$

Problem 4 (10 pts)
Consider the pressurized water reactor in the figure below.



a. Identify the components of the reactor core labelled a, b, c and d and briefly describe their function. (4 pts)
b. Explain why it is necessary to use enriched uranium in this type of reactor. (2 pts)
c. What would you change inside this reactor if you were to use natural uranium? (4 pts)

Solution

a. The components labelled a, b, c and d correspond to the control rods, the fuel rods, steam generator and pressuriser, respectively.
The control rods are used to control the power of a nuclear reactor: removing or inserting control rods into the reactor core, the neutron flux is increased or decreased, which influences the speed of the chain reaction.
The fuel rods contain enriched uranium shaped into fuel pellets, which are stacked into cladding that absorbs the fission fragments and prevents them from escaping into the coolant. The fuel rods are bundled together in a specific structure where the coolant can flow.
The steam generator is a heat exchanger used to convert water into steam from heat produced in the nuclear reactor core.
The pressurizer is used to control the coolant pressure in the primary reactor cooling system to the level that boiling is effectively suppressed inside the reactor vessel.

b. A PWR is a thermal reactor, in which the moderation of neutrons is achieved by collisions between neutron and water molecules (in particular hydrogen). Water has a relatively large absorption for neutrons, which means that some of neutrons will be lost and not participate in fission reactions. This can be compensated by enriching the fuel, that is increasing the amount of fissile material (3-5%) ^{235}U.

c. If you were to replace the enriched uranium by natural uranium, you would need to use heavy water as moderator (and coolant), as in the CANDU reactor.

Avogadro number $N_A = 6.022 \times 10^{23} mol^{-1}$

Cross section $1\ b = 10^{-24} cm^2$

Number density

$$N\ [cm^{-3}] = \frac{\rho\ [g/cm^3]\ N_A\ [mol^{-1}]}{M\ [u = g/mol]}$$

Reaction rate $R = \Sigma\phi = N\sigma\phi$

Total average power of a reactor

$$P[W = J/s] = E_f[J] \times R_f[cm^{-3}s^{-1}] \times V[cm^3]$$

Neutron multiplication factor

$$k = \frac{\nu\ \Sigma_f}{\Sigma_a + DB^2}$$

Conversion/equivalence

$1\ MeV = 1.602\ 10^{-13}\ J$

$1\ W = 1\ J/s$

$E_f = 200\ MeV$

Buckling and neutron flux for different reactor geometries

Geometry	**Dimensions**	**Buckling B^2**	**Flux**	**A**
Infinite slab	Thickness a	$\left(\frac{\pi}{a}\right)^2$	$A\cos\left(\frac{\pi}{a}x\right)$	$1.57 \times \frac{P}{a\,E_f\,\Sigma_f}$
Rectangular parallepiped	a x b x c	$\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2+\left(\frac{\pi}{c}\right)^2$	$A\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{c}z\right)$	$3.87 \times \frac{P}{V\,E_f\,\Sigma_f}$
Infinite cylinder	Radius R	$\left(\frac{2.405}{R}\right)^2$	$AJ_0\left(\frac{2.405\,r}{R}\right)$	$0.738 \times \frac{P}{R^2 E_f \Sigma_f}$
Finite cylinder	Radius R, height H	$\left(\frac{2.405}{R}\right)^2+\left(\frac{\pi}{H}\right)^2$	$AJ_0\left(\frac{2.405\,r}{R}\right)\cos\left(\frac{\pi\,z}{H}\right)$	$3.63 \times \frac{P}{V\,E_f\,\Sigma_f}$
Sphere	Radius R	$\left(\frac{\pi}{R}\right)^2$	$A\,\frac{1}{r}\sin\left(\frac{\pi\,r}{R}\right)$	$\frac{P}{4R^2\,E_f\,\Sigma_f}$